



ST. FRANCIS DE SALES COLLEGE

A FRANSALIAN INSTITUTE OF HIGHER EDUCATION **AUTONOMOUS**

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END SEMESTER EXAMINATION – DECEMBER 2024

MATHEMATICS – I SEMESTER BSC

24BSC13A/B - MATHEMATICS - I

Time: 3 Hours

Max. Marks: 80

Instruction: Answers should be written completely in English

SECTION – A

I. Answer any ten of the following:

(10×2=20)

1. Define rank of a matrix.
2. Find the rank of the matrix by reducing to row reduced echelon form $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -1 & 4 \end{bmatrix}$.
3. Find the eigen values of the matrix $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.
4. Find the n^{th} derivatives of the function $y = \log(4x + 3)$.
5. If $u = x^3 + y^3 - 3axy$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
6. State Euler's theorem for a homogeneous function.
7. Show that $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x \, dx = \frac{3\pi}{512}$.
8. Evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x \, dx$.
9. Write the formula to find the volume generated by the curve $y = f(x)$ revolving about the y -axis.
10. Find the equation of the sphere whose centre is $(1, -2, 3)$ and radius is 4 units.
11. Write the equation of the right circular cylinder with radius 2 units and the axis is the line $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$.
12. Write the equations of hyperboloid of one sheet with figure.



SECTION -- B

II. Answer any three of the following

(3×5=15)

13. Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 2 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$.

14. Test the following system for consistency and hence solve:

$$3x + y + z = 8; -x + y - 2z = -5; x + y + z = 6; -2x + 2y - 3z = -7.$$

15. For what values of λ and μ , the equations

$$x + 2y + z = 8; 2x + y + 3z = 13; 3x + 4y - \lambda z = \mu$$

have (i) no solution, (ii) unique solution and (iii) infinitely many solutions.

16. Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

17. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and find it's inverse.

SECTION – C

III. Answer any three of the following

(3×5=15)

18. Find the n^{th} derivative of $\sin^2 x \cos^3 x$.

19. If $y = (\sin^{-1} x)^2$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

20. If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

21. If $u = 2xy$ and $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$ prove that $\frac{\partial(u,v)}{\partial(r,\theta)} = -4r^3$.

22. If $u + v = e^x \sin y$, $u - v = e^x \cos y$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

SECTION – D

IV. Answer any three of the following

(3×5=15)

23. Obtain the reduction formula for $\int \sin^n x \, dx$, where n is a positive integer.

24. Evaluate $\int_0^a x^3 \sqrt{ax - x^2} \, dx$.



25. Evaluate $\int_0^\pi \frac{\sin^4 x}{(1+\cos x)^2} dx$.

26. Find the length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

27. Find the surface area of a hemisphere of the radius 'a'.

SECTION - E

V. Answer any three of the following

(3×5=15)

28. Find the equation of the sphere passing through the following points

(0, 0, 0), (0, 1, -1), (-1, 2, 0) and (1, 2, 3).

29. Find the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which is parallel to the plane $2x + 2y - z = 0$.

30. Derive the equation of right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.

31. Find the right circular cylinder generated by revolving the line

$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$ about the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+5}{-1}$.

32. Explain the equation of hyperboloid of one sheet with properties.

