



# ST. FRANCIS DE SALES COLLEGE

A FRANSALIAN INSTITUTE OF HIGHER EDUCATION AUTONOMOUS

NAAC A GRADE • AFFILIATED TO BANGALORE UNIVERSITY • AICTE APPROVED • 2(F) & 12 (B) RECOGNITION OF UGC • ISO 9001:2015 CERTIFIED  
Electronics City P.O. Bengaluru - 560 100, Karnataka, INDIA • (+91) 8088140679 • [pr@stfcollge.in](mailto:pr@stfcollge.in) • [www.stfcollge.in](http://www.stfcollge.in)

## END SEMESTER EXAMINATION – DECEMBER 2024

### MATHEMATICS – I SEMESTER BSC

### 24BSC13A/B - MATHEMATICS - I

Time: 3 Hours

Max. Marks: 80

Instruction: *Answers should be written completely in English*

#### SECTION – A

##### I. Answer any ten of the following:

(10×2=20)

1. Define rank of a matrix.
2. Find the rank of the matrix by reducing to row reduced echelon form  $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -1 & 4 \end{bmatrix}$ .
3. Find the eigen values of the matrix  $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$ .
4. Find the  $n^{\text{th}}$  derivatives of the function  $y = \log(4x + 3)$ .
5. If  $u = x^3 + y^3 - 3axy$ , show that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ .
6. State Eulers's theorem for a homogeneous function.
7. Show that  $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x \, dx = \frac{3\pi}{512}$ .
8. Evaluate  $\int_0^{\frac{\pi}{2}} \cos^8 x \, dx$ .
9. Write the formula to find the volume generated by the curve  $y = f(x)$  revolving about the  $y$ -axis.
10. Find the equation of the sphere whose centre is  $(1, -2, 3)$  and radius is 4 units.
11. Write the equation of the right circular cylinder with radius 2 units and the axis is the line  $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ .
12. Write the equations of hyperboloid of one sheet with figure.



## SECTION - B

**II. Answer any three of the following** (3×5=15)

13. Find the rank of the matrix by reducing it to normal form  $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 2 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ .

14. Test the following system for consistency and hence solve:

$$3x + y + z = 8; -x + y - 2z = -5; x + y + z = 6; -2x + 2y - 3z = -7.$$

15. For what values of  $\lambda$  and  $\mu$ , the equations

$$x + 2y + z = 8; 2x + y + 3z = 13; 3x + 4y - \lambda z = \mu$$

have (i) no solution, (ii) unique solution and (iii) infinitely many solutions.

16. Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

17. Verify Caley – Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and find it's inverse.

## SECTION - C

**III. Answer any three of the following**

(3×5=15)

18. Find the  $n^{\text{th}}$  derivative of  $\sin^2 x \cos^3 x$ .

19. If  $y = (\sin^{-1} x)^2$  show that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ .

20. If  $z = \sin(ax + y) + \cos(ax - y)$  prove that  $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$ .

21. If  $u = 2xy$  and  $v = x^2 - y^2$  and  $x = r\cos\theta, y = r\sin\theta$  prove that  $\frac{\partial(u,v)}{\partial(r,\theta)} = -4r^3$ .

22. If  $u + v = e^x \sin y, u - v = e^x \cos y$  find  $\frac{\partial(u,v)}{\partial(x,y)}$ .

## SECTION - D

**IV. Answer any three of the following**

(3×5=15)

23. Obtain the reduction formula for  $\int \sin^n x \, dx$ , where  $n$  is a positive integer.

24. Evaluate  $\int_0^a x^3 \sqrt{ax - x^2} \, dx$ .



25. Evaluate  $\int_0^\pi \frac{\sin^4 x}{(1+\cos x)^2} dx$ .

26. Find the length of the asteroid  $x^{2/3} + y^{2/3} = a^{2/3}$ .

27. Find the surface area of a hemisphere of the radius 'a'.

## SECTION - E

### V. Answer any three of the following

(3×5=15)

28. Find the equation of the sphere passing through the following points

(0, 0, 0), (0, 1, -1), (-1, 2, 0) and (1, 2, 3).

29. Find the tangent plane to the sphere  $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$  which is parallel to the plane  $2x + 2y - z = 0$ .

30. Derive the equation of right circular cone in its standard form  $x^2 + y^2 = z^2 \tan^2 \alpha$ .

31. Find the right circular cylinder generated by revolving the line

$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$  about the line  $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+5}{-1}$ .

32. Explain the equation of hyperboloid of one sheet with properties.

