ST. FRANCIS DE SALES COLLEGE

A FRANSALIAN INSTITUTE OF HIGHER EDUCATION **AUTONOMOUS**

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Electronics City P.O., Bengaluru - 560 100, Karnataka, INDIA (+91) 8088140679 pro@sfscollege.in www.sfscollege.in

END SEMESTER EXAMINATION – NOV/DEC 2025

MATHEMATICS – III SEMESTER BSC

25BSC33A/B: MATHEMATICS III

Time: 3 Hours **Max. Marks: 80**

Instruction: *Answer should be written completely in English.*

SECTION - A

Answer any **FIVE** questions. Each question carries **TWO** marks. **(5X 2=10)**

1. Show that f: G→G' defined by G'=$\{2^n / n\epsilon z\}$is homomorphism.
2. Show that the homomorphic image of an abelian group is abelian.
3. What is monotonic decreasing sequence?
4. Define Convergent of a sequence with an example.
5. Show that the series $\sum(-1)^n$is Oscillates finitely.
6. Test the convergence of the series $\sum sin\frac{1}{n}$.
7. If $\varphi(x,y,z)=3x^2 y-y^3z^2$ find $\nabla\varphi$ at (1,-2,-1).
8. Show that Curl (grad φ)=0.

SECTION -B

Answer any **SIX** questions. Each question carries **FIVE** marks. **(6X 5=30)**

9. If G= {1, -1, i, -i} is a group under multiplication and H= {1, -1} is a subgroup of G. Show that H is a normal subgroup of G.
10. Show that f: G→G' defined by f(x)=cosx+isinx is a homomorphism. Find Kernel. Is f an isomorphism. Where G and G' are real numbers w.r.t addition and multiplication respectively.
11. Show that the sequence $\left(1-\frac{1}{n}\right)$ is a monotonic decreasing sequence.
12. Let $\{x_n\}$ and $\{y_n\}$ be two convergent sequences and let $\log_{n\to\infty} x_n = l$ and $\log_{n\to\infty} y_n = m$. Then $\log_{n\to\infty}(x_n + y_n) = l + m$.
13. Sum to infinity the series $\frac{1}{6}+\frac{1.4}{6.12}+\frac{1.4.7}{6.12.18}+\cdots$

14. Discuss the convergence of the series using Leibnitz Rule

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n-1}.$$

15. Prove that $\nabla^2 \left(\frac{1}{r}\right)$=0.

16. If $\vec{u} = x^2 i + y^2 j + z^2 k$ *and* $\vec{v} = yzi + zxj + xyk$ show that $\vec{u} \times \vec{v}$ is a solenoidal vector.

SECTION – C

Answer any **FIVE** questions. Each question carries **EIGHT** marks. **(5X8=40)**

17. Define homomorphism with an example. If f: G→G be a homomorphism and H is a cyclic subgroup of G, then show that f(H) is again cyclic subgroup of G.

18. Let (z, +) be the additive group of all integers and H= {1, -1} be the multiplicative group. Define f: Z→H by f(z)=$\begin{cases} 1 \text{ if } z \text{ is even} \\ -1 \text{ if } z \text{ is odd} \end{cases}$. Show that f is a homomorphism. What is Kerf? Is f an isomorphism.

19. State and prove Cauchy's principle of Convergence.

20. Test the convergence of the following sequence.

    i. $\frac{(n+1)^{n+1}}{n^n}$ iii. $\left(\frac{n+1}{n-1}\right)^n$

    ii. $\left(1-\frac{a}{n}\right)^{\frac{n}{b}}$ iv. $\left(\frac{n+1}{n}\right)^{\frac{2n^2}{n+1}}$

21. The geometric series $\sum_{n\to 0}^{\infty} x^n$.
    i. Converges if $|x| < 1$.
    ii. Diverges if $x \geq 1$.
    iii. Oscillates if $x \leq -1$.

22. Let $\sum u_n$ and $\sum v_n$ be two series of positive terms, such that
    i. $\sum v_n$ is divergent.
    ii. $u_n \geq k.v_n$ for all values of n, except perhaps for the first few finite numbers of terms.

    Then $\sum u_n$ is also divergent.

23. Find the value of a and b so that the surface $5x^2$-2yz-9x=0 may cut the surface $ax^2+by^3=4$ orthogonally at (1, -1, 2).

24. Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find $\boldsymbol{\varphi}$ such that $\vec{F} = \nabla\boldsymbol{\varphi}$.