



ST. FRANCIS DE SALES COLLEGE

A FRANSALIAN INSTITUTE OF HIGHER EDUCATION **AUTONOMOUS**

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END SEMESTER EXAMINATION – APRIL/MAY 2026

MATHEMATICS - II SEMESTER BSC

25BSC23A: MATHEMATICS II

Time: 3 Hours.

Max. Marks: 80

Instruction: Answer should be written completely in English.

SECTION - A

Answer any **FIVE** questions. Each question carries **TWO** marks.

(5X2=10)

1. If $a * b = \frac{3ab}{7}$ then prove that $*$ is associative.
2. Find the number of generators of the cyclic group of order 24.
3. Find $\frac{ds}{d\theta}$ for the curve $r = a(1 + \cos\theta)$.
4. Find the asymptote parallel to the y axes for the curve $y^3 - x^2y = x^2 + 1$.
5. State Lagrange's mean value theorem.
6. Evaluate $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$ by using L Hospital's rule.
7. Find the integrating factor of $\frac{dy}{dx} + \frac{2}{x}y = x^3$.
8. Show that the equation is $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ is exact.

SECTION - B

Answer any **SIX** questions. Each question carries **FIVE** marks.

(6X5=30)

9. Show that the fourth roots of unity form an abelian group under multiplication.
10. If an element 'a' of a group G is of order n and e is the identity element in G, then for some positive integer m, $a^m = e$ if and only if n is a divisor of m.
11. Find the angle between the radius vector and the tangent for the curve $r = a(1 + \cos\theta)$.
Also find the slope of the curve at $\theta = \frac{\pi}{6}$.
12. Show that the pedal equation of the Lemniscate $r^2 = a^2 \cos 2\theta$ is $a^2 p = r^3$.
13. State and prove Cauchy's mean value theorem.
14. Expand $\log(1 + \sin x)$ by using Maclaurin's series up to the term containing x^4 .
15. Verify for exactness and solve $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4) dy = 0$.
16. Obtain the orthogonal trajectories of the family of the circles $x^2 + y^2 = c^2$.



SECTION - C

Answer any **FIVE** questions. Each question carries **EIGHT** marks.

(5X8=40)

17. a) In a Group G show that $o(a) = o(a^{-1})$.
b) Find the order of each element of the group $G = \{0, 1, 2, 3, 4, 5\}$ w.r.t addition modulo 6.
18. a) If a is a generator of a cyclic group G , then prove that a^{-1} is also a generator.
b) Find all the right and left cosets of the subgroup $H = \{0, 2, 4\}$ of the group $(\mathbb{Z}_6, +_6)$.
19. Find the angle of intersection for the curves $r = a(1 - \cos\theta)$; $r = 2a\cos\theta$.
20. a) Find $\frac{ds}{dx}$ for the curves $x = a(t + \sin t)$, $y = a(1 - \cos t)$.
b) Find the radius of curvature at any point on the curve $xy = c^2$.
21. a) Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$.
b) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at the point $(1, 1)$.
22. a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ by using L Hospital's rule.
b) Evaluate $\lim_{x \rightarrow 0} x \log(\tan x)$ by using L Hospital's rule.
23. Solve $x^2 p^2 + xyp - 6y^2 = 0$.
24. Find the general and singular solution of $(p - 1)e^{3x} + p^3 e^{2y} = 0$ by using the substitution $u = e^x$ and $v = e^y$.

