



ST. FRANCIS DE SALES COLLEGE

A FRANSALIAN INSTITUTE OF HIGHER EDUCATION **AUTONOMOUS**

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END SEMESTER EXAMINATION – NOV/DEC 2025

MATHEMATICS – I SEMESTER BSC (Repeaters)

24BSC13A/B: MATHEMATICS I

Time: 3 Hours

Max. Marks: 80

Instruction: Answer should be written completely in English.

SECTION - A

Answer any **TEN** questions. Each question carries **TWO** marks.

(10X2=20)

1. Find the value of k such that the rank of a matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{pmatrix}$ is two.
2. Find the eigen values of $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.
3. State the Caley-Hamilton theorem.
4. Write the formula of n th derivative of e^{ax} .
5. Find the n^{th} derivative of $\frac{1}{3x-1}$.
6. If $z = \log(x^2+y^2)$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
7. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x \, dx$.
8. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x \, dx$.
9. Write the formula to find the surface area of the solid generated by the revolution of the area bounded by the curve $y=f(x)$ about the x -axis.
10. Find the equation of the sphere whose centre is $(2,-3,4)$ and radius 5.
11. If a right circular cone has three mutually perpendicular generators, show that the semi vertical angle is $\tan^{-1}\sqrt{2}$.
12. Find the equation of the right circular cylinder given that the radius of its normal section is 4 and the axis passes through the points $A = (1,-2,3)$ and $B = (3,-1,1)$.

SECTION - B

Answer any **THREE** questions. Each question carries **FIVE** marks.

(3X5=15)

13. By reducing to echelon form find the rank of a matrix $A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$.
14. Solve the system $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$.
15. Investigate for what values of λ, μ the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.



16. Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$.
17. Using Caley-Hamilton theorem find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$.

SECTION – C

Answer any **THREE** questions. Each question carries **FIVE** marks.

(3X5=15)

18. Find the n^{th} derivative of $\frac{x-1}{(x-2)^2(x+2)}$.
19. If $y = \cos(m \cos^{-1} x)$ prove that $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 - m^2) y_n = 0$.
20. If $u=f(x,y)$ is a homogeneous function of x and y with degree n , then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$.
21. If $z = \sin(ax+y) + \cos(ax-y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.
22. If $x = r \cos \theta$, $y = r \sin \theta$, find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$. Also verify that $JJ' = 1$.

SECTION – D

Answer any **THREE** questions. Each question carries **FIVE** marks.

(3X5=15)

23. Obtain the reduction formula for $\int \sin^n x \, dx$, where n is a positive integer.
24. Evaluate $\int_0^\pi \frac{\sin^4 \theta}{(1-\cos \theta)^2} d\theta$.
25. Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$, where m and n are positive integers.
26. Find the arc length of the parabola $y^2 = 4ax$ cut off by the latus rectum.
27. Find the surface area of a hemisphere of radius 'a'.

SECTION – E

Answer any **THREE** questions. Each question carries **FIVE** marks.

(3X5=15)

28. Find the equation of the sphere having the points $(2, 1, -3)$ and $(1, -2, 4)$ as the ends of a diameter, also find its centre and radius.
29. Find the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which is parallel to the plane $2x + 2y - z = 0$.
30. Obtain the equation of the right circular cone with vertex at $(3, 1, 2)$, axis has direction ratios $(1, 2, 3)$ and semi vertical angle $\cos^{-1}(\frac{1}{\sqrt{7}})$.
31. Find the equation of a right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.
32. Explain the equation of ellipsoid with properties.

