PG – 161

III Semester M.Sc. Degree Examination, March/April 2021
(CBCS – Y2K17 Scheme)
MATHEMATICS
M 301 T : Differential Geometry

Time : 3 Hours

Max. Marks : 70

Instructions : *Answer **any 5 full** questions from the following. **All** questions carry **equal** marks.*

1. a) If $V = xU_1 + yU_3$ and $W = 2x^2U_2 - U_3$, then compute $W - xV$ and find its value at $p = (-1, 0, 2)$.

   b) Let $V_p : V = (2, 1, -3)$, $P = (2, 0, -1)$ and let $f = y^2z$, $g = e^x \cos y$. Then compute (i) $V_p[f]$, (ii) $V_p[g]$.

   c) For vector fields V and W on E^3, if $V[f] = W[f]$, for all differentiable function f on E^3, then prove that $V = W$.

   (4+6+4)

2. a) Explain reparametrization of a curve in E^3. Reparametrize the curve $\alpha(t) = (e^t, e^{-t}, \sqrt{2}\,t)$ using $h(s) = \log s$ and verify the formula $\beta'(s) = \frac{dh}{ds}(s)\,\alpha'(h(s))$.

   b) Derive the Frenet formulae for a unit speed curve.

   (7+7)

3. a) Show that the curve $\alpha(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$ is a cylindrical helix.

   b) Let $f = (z^2 - 1)\,dx - dy + x^2dz$ and $V_p = (1, 2, -3)$, $p = (0, -2, 1)$. Then find df and evaluate $V_p[f]$.

   c) If $W = \sum w_iU_i$ and V is a vector field on E^3, then prove that $\nabla_v W = \sum_{i=1}^{3} V[w_i]U_i$. Use it to compute $\nabla_v W$ for $W = x^2U_1 + yzU_3$.

   (5+4+5)

4. a) Compute the derivative map $F_{*p}(V_p)$ for the mapping $F(x, y, z) = (x \cos y, x \sin y, z)$ and $V_p : V = (2, -1, 3)$, $p = (0, 0, 0)$.

   b) Verify the structural equations for the cylindrical frame field.

   c) Let F be an isometry of E^3 such that $F(0) = 0$. Then prove that F is an orthogonal transformation.

   (4+6+4)

P.T.O.

5. a) Which of the following is a patch ?

   i) $X(u, v) = (u + v, u - v, uv)$

   ii) $X(u, v) = (u^2, uv, v^2), u > 0$.

   b) Define a surface. Use the definition to show that unit sphere in E^3 is a surface.

   c) Show that a surface of revolution is a surface. **(4+5+5)**

6. a) Show that a mapping $X : D \to E^3$ is regular if and only if the u, v – partial derivatives $X_u(d)$, $X_v(d)$ are linearly independent for all $d \in D$, where $D \subseteq E^3$.

   b) Explain parametrization of a region and obtain parametrization of surface of revolution.

   c) Define pull back function for P-form (P = 1,2). Show that (i) $F^*(\xi \wedge \eta) = F^*\xi \wedge F^*\eta$ (ii) $F^*(d\xi) = d(F^*\xi)$. **(5+5+4)**

7. a) Show that the shape operator S_p is a linear operator and obtain shape operator of a cylinder.

   b) Let α be a curve in $M \subseteq E^3$. If U is a unit normal of M restricted to the curve α, show that $S(\alpha').\alpha' = \alpha''.U$.

   c) If p is a non-umbilic point $K_1 \neq K_2$, then there are exactly two principal directions and these are orthogonal. Further, if e_1 and e_2 are principal vectors in these directions then $S(e_1) = k_1 e_1$, $S(e_2) = k_2 e_2$. **(5+3+6)**

8. a) Show that for any patch X in M in E^3, $l = S(X_u)\cdot X_u = U\cdot X_{uu}$, $m = S(X_u)\cdot X_v = U\cdot X_{uv}$, $n = S(X_v)\cdot X_v = U.X_{vv}$. Compute the Gaussian and mean curvatures of $X(u, v) = (u \cos v, u \sin v, bv)$, $b \neq 0$.

   b) Let α be a regular curve in $M \subset E^3$, and let U be a unit normal vector field restricted to α. Then prove the followings.

   i) the curve α is principal if and only if U' and α' are collinear at each point.

   ii) the principal curvature of principal curve α in the direction of α' is $\frac{(\alpha''.U)}{\alpha'.\alpha'}$.

   c) Determine the geodesics of (i) sphere (ii) cylinder. **(7+3+4)**

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