PG – 162

III Semester M.Sc. Degree Examination, March/April 2021
(CBCS – Y2K17)
MATHEMATICS
M302T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

Instructions : 1) *Answer* ***any five full*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) The x_i' system is obtained by rotating the x_i system about the x_3 axis through an angle θ in the sense of the right handed screw, then find the transformation matrix. Further, if a point p has co-ordinates (1, 1, 1) in the x_i system, then find its co-ordinates in the x_i' system.

   b) Define scalar invariants. Let $\vec{a}$ and $\vec{b}$ be vectors with components a_i and b_i respectively. Let A be a tensor with components a_{ij}, then show that $a_i b_i$ and a_{ii} are scalar invariants. **(7+7)**

2. a) Define symmetric and skew symmetric tensors. Show that for a skew tensor A, there exists a unique dual vector $\vec{w}$ such that $A\vec{u} = \vec{w} \times \vec{u}$ for every vector $\vec{u}$.

   b) State and prove divergence theorem for a tensor. **(7+7)**

3. a) For the flow defined by the velocity field $\vec{v} = (1+at)\, e_1 + x_1 e_2$ where 'a' is constant, find the path lines and stream lines.

   b) Establish Reynolds transport formula in its standard form.

   c) Explain briefly the concept of stress components. **(5+5+4)**

4. a) Obtain the conservation of energy in its standard form.

   b) Establish the Euler's equation of motion in its most general form. **(10+4)**

5. a) Derive Navier-Stokes equation for a compressible fluid in its standard form and write the same in the component form.

   b) Obtain the exact solution of the Navier-Stokes equation for the couetle flow problem. Further, calculate the

      i) Maximum velocity

      ii) Average velocity and

      iii) Shearing stress on the wall for the same. **(7+7)**

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6. a) For the steady and slow flow of an incompressible viscous fluid under zero body force, show that the pressure is a harmonic function.

   b) Obtain the exact solution of the Navier-Stokes equation for the Hagen-Poiseuille flow problem. Further, calculate,

      i) Maximum velocity

      ii) Average velocity and

      iii) Shearing stress for the same. **(4+10)**

7. Establish the Stokes second problem and solve it. **14**

8. a) State and prove Milne-Thomson circle theorem.

   b) In a two-dimensional flow field $\psi = xy$. Show that the flow is irrotational. Find the velocity potential and verify that ϕ and ψ satisfy the Laplacian equation. Also, find stream line and potential line. **(7+7)**
