PG – 165

Third Semester M.Sc. Degree Examination, March/April 2021
(CBCS – Y2K17)
MATHEMATICS
M305 T : Numerical Analysis – II

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Determine four non-zero solutions of $\frac{du}{dx} = x^2 + u^2$, $u(0) = 1$ using Taylor series method at $x = 0.4$.

   b) Derive the Runge-Kutta method of order two and four for the solution of $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. **(6+8)**

2. a) Discuss the stability of Runge-Kutta of order two and four methods.

   b) Solve the system of equations :

   $$\frac{dy}{dx} = -3y + 2z, \; y(0) = 0$$

   $$\frac{dz}{dx} = 3y - 4z, \; z(0) = \frac{1}{2}$$

   at $x = 0.2$ and $x = 0.4$ using an appropriate numerical method. **(7+7)**

3. a) Derive the three-step Adams-Bashforth method for $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

   b) Using four-step Adams-Bashforth method, for

   $\frac{dy}{dx} = xe^{3x} - 2y$, $y(0) = 0$, determine the solution at $x = 0.2$. **(7+7)**

4. Describe the shooting method for the solution of higher order differential equations. Hence, solve $\frac{d^2y}{dx^2} + \frac{1}{8}y\frac{dy}{dx} = \frac{1}{4}x^3 + 4$ with $y(1) = 17$, $y(3) = \frac{43}{3}$.

   Take the initial guess $\frac{dy}{dx}(0) = -16.33$. **14**

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5. Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x^2 - 1$, $|x| \leq 1$, $|y| \leq 1$ subjected to the condition u = 0 on the square, make use of the symmetry of the square. Take $\Delta x = \Delta y = 0.5$. **14**

6. a) Discuss the stability of the Schmidt method.

   b) Solve the heat equation $U_t = U_{xx}$, with conditions $U(x, 0) = 1$, $U_x(0, t) = 0 = U_x(1, t)$. Using the Crank-Nicolson scheme. Take $\Delta x = \frac{1}{3}$, $\Delta t = \frac{1}{27}$. Perform four iterations of Gauss-Seidel method. **(5+9)**

7. a) Solve the wave equation $U_{tt} = U_{xx}$, with conditions $U(x, 0) = \frac{1}{8} \sin \pi x$, $U_t(x, 0) = 0$, $U(0, t) = 0 = U(1, t)$, using an implicit finite difference scheme. Take $\Delta x = 0.25$, $\Delta t = 0.1$.

   b) Derive the first Lees alternating direction implicit method applied to two-dimensional wave equation. **(7+7)**

8. Find a solution for $U_{tt} = U_{xx} + U_{yy}$, $0 \leq x, y \leq 1$, $t \geq 0$ with $U(x, y, 0) = \sin \pi x \sin \pi y$, $U_t(x, y, 0) = 0$, U = 0 on the boundary. Perform one time integration using the first Lees ADI method. Take $\Delta x = \frac{1}{3}$, $\Delta y = \frac{1}{3}$, $\Delta t = \frac{1}{9}$. **14**
