PG – 140

Third Semester M.Sc. Degree Examination, April/May 2022
(CBCS – Y2K17/Y2K14 Scheme)
MATHEMATICS
M301T : Differential Geometry

Time : 3 Hours Max. Marks : 70

Instructions : 1) *Answer **any five full** questions.*
2) ***All** questions carry **equal** marks.*

1. a) Define directional derivative of a function. Compute $V_p[f]$ for $f = x^2yz$, $v = (1, 0, -3)$, $p = (1, 1, 0)$.

b) If $h(S) = \log S$ on J, $S > 0$. Reparametrise $\alpha(t) = (e^t, e^{-t}, \sqrt{2}t)$ using h. Verify the result $\beta'(S) = \alpha'(h(S))\frac{dh}{ds}(S)$.

c) Let α be a Curve in E^3. Then prove that $\alpha'(t)[f] = \frac{d}{dt}(f \circ \alpha)(t)$. **(4+7+3)**

2. a) In each case compute differential df of f and find its directional derivative $V_p[f]$ for V_p.

i) $f = xy^2 - yz^2$

ii) $f = xe^{yz}$.

b) If ϕ and ψ are 1– forms on E^3 then prove that $d(\phi \wedge \psi) = d\phi \wedge \psi - \phi \wedge d\psi$.

c) Let $F : E^3 \to E^3$ be defined by $F(x, y, z) = (x \cos y, x \sin y, z)$. Compute FV_p for $V_p : V = (2, -1, 3)$, $p = (0, 0, 1)$. **(4+5+5)**

3. a) Compute the Frenet apparatus T, N, B, J, K for the curve $\beta(s) = \left(a\cos {s}/{c}, a\sin {s}/{c}, b\, {s}/{c}\right)$, where $C = \sqrt{a^2 + b^2}$.

b) Show that

i) $\nabla_{aV_p + bW_p} Y = a\nabla_{V_p} Y + b\nabla_{W_p} Y$.

ii) $V_p[Y.Z] = \nabla_{V_p} Y.Z(p) + Y(p) . \nabla_{V_p} Z$.

for any tangent vectors V_p, W_p to E^3 at p, vector fields Y, Z on E^3 any real numbers a and b.

c) Compute $\nabla_{V_p} W$ for $W = x^2 U_1 + yzU_3$ any $V_p : V = (-1, 0, 2)$, $P = (2, 1, 3)$. **(5+6+3)**

P.T.O.

4. a) Derive Cylindrical frame field and find its connection forms.

   b) If F is an isometry of E^3, then prove that there exist a unique translation T and a unique orthogonal transformation C such that F = TC. **(6+8)**

5. a) Let f be a real valued differentiable function on a non-empty open set D of E^2. Then show that the function $x : D \to E^3$ satisfying $x(u, v) = (u, v, f(u, v))$ is a proper patch in E^3.

   b) Let g be a differentiable real valued function on E^3 and C a number. Show that the subset M : g (x, y, z) = C of E^3 is a surface. If the differential dg is not zero at any point of M. **(6+8)**

6. a) Let p be a point of a surface M in E^3 and let X be a patch in M such that $X(u_0, v_0) = p$. Show that a tangent vector V_p to E^3 at p is tangent to M if and only if V_p can be written as a linear combination of $X_u(u_0, v_0)$ and $X_v(u_0, v_0)$.

   b) Obtain the parametrisation of cylinder.

   c) If ξ and η are 1-forms on N and $F : M \to N$ is a mapping then show that
      i) $F^*(\xi + \eta) = F^*\xi + F^*\eta$.
      ii) $F^*(\xi \wedge \eta) = F^*\xi \wedge F^*\eta$. **(5+3+6)**

7. a) Obtain the shape operator of a Sphere of radius r.

   b) Define principal curvatures. Show that, if p is an umbilic point of a surface M in E^3 then the shape operator S at p in just a scalar multiplication by $K = K_1 = K_2$.

   c) With usual notations prove $K = K_1K_2$, $H = \frac{1}{2}(K_1 + K_2)$. **(4+7+3)**

8. a) If X is a patch in a Surface M in E^3 then show that the Gaussion curvature K and mean curvature H of M are given by $K = \frac{ln - m^2}{EG - F^2}$, $H = \frac{Gl + En - 2Fm}{2(EG - F^2)}$.

   b) Compute K, H and hence K_1, K_2 for X (u, v) = (ucosv, usinv, bv), $b \neq 0$.

   c) Determine the geodesics of a plane. **(5+7+2)**

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