III Semester M.Sc. Degree Examination, April/May 2022
(CBCS – Y2K17/Y2K14 Scheme)
MATHEMATICS
M302T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five full** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Define Levi-Civita ε-symbol. Complete the matrix

$$\left[\alpha_{ij}\right]=\begin{bmatrix} \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{2\sqrt{2}} & \frac{-1}{2\sqrt{2}} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}.$$

b) Define :

i) Tensor product of vectors

ii) Cartesian tensor of second order

iii) Isotropic tensor.

c) Let $\vec{a}$ and $\vec{b}$ be two vectors with components a_i and b_i respectively. Let A be a tensor with components a_{ij}. Then show that $a_i b_i$ and a_{ii} are scalar invariants. **(6+3+5)**

2. a) If a_{ij} are components of an isotropic tensor of second order, then show that $a_{ij} = \alpha\delta_{ij}$ for some scalar α.

b) Let A be a skew tensor, then show that there exists a unique vector $\vec{w}$ such that $A\vec{u} = \vec{w} \times \vec{u}$ for every vector $\vec{u}$. **(8+6)**

3. a) Let Q(t) is an orthogonal tensor, then show that $\left(\frac{dQ}{dt}\right)Q^T$ is skew tensor.

b) Define :

i) Curl of a vector field.

ii) Curl of a tensor field.

c) State and prove Stokes theorem for a tensor. **(4+4+6)**

P.T.O.

4. a) Define path lines, stream lines, vertex lines and field lines.

   b) Explain briefly the concept of principal stresses.

   c) Establish the Reynold's transport formula in its standard form. **(4+4+6)**

5. a) Obtain the conservation of mass in its standard form and obtain an important consequence of the same.

   b) The stress components of a continuum in equilibrium are given by
   $\tau_{11} = x_1^2, \tau_{22} = x_2^2, \tau_{33} = x_1^2 + x_2^2, \tau_{12} = \tau_{21} = 2x_1x_2, \tau_{23} = \tau_{32} = 0, \tau_{31} = \tau_{13} = 0.$
   Find the body force that must be acting on the continuum. **(9+5)**

6. a) Obtain the exact solution of the Navier Stokes equation for the steady flow between two rotating concentric circular cylinders. Further explain mathematically

      i) Radial pressure in peripheral motion

      ii) Shearing stress for the same.

   b) Find the pressure distribution such that velocity field is given by $\vec{v} = k(x^2 - y^2)i - 2kxy\, j$ (k is constant) satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body force. **(8+6)**

7. a) Obtain an expression for the rate of decrease in kinetic energy due to viscosity of an incompressible fluid in a solid surface at rest.

   b) Discuss the flow governed by the complex potential $w = \frac{2f}{z}$, where f is constant. **(7+7)**

8. a) Show that w = – uz – m/n(z) + m/n (z – z_0) represents a system with a uniform flow, where a source of strength 'm' at z = 0 and a sink of strength 'm' at z = z_0.

   b) State and prove Milne-Thomson circle theorem. **(7+7)**