III Semester M.Sc. Degree Examination, April/May 2022
(Y2K17/Y2K14) (CBCS)
MATHEMATICS
M305T : Numerical Analysis – II

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Solve by three iterations of the Picard's method for the initial value problem $\frac{d^2y}{dx^2} + xy = 0$; $y(0) = 1$, $\frac{dy(0)}{dx} = 0$ and estimate y(0.05).

   b) Solve $\frac{dy}{dx} = y + e^x$; y(0) = 0, find y(0.2) taking h = 0.2 by modified Euler's method. **(8+6)**

2. a) By making use of the classical, explicit Runge-Kutta method of fourth order, solve

   $\frac{dy}{dx} = 3y + 2z$; y (0) = 1

   $\frac{dz}{dx} = 3y - 4z$; z(0) = 0 and compute y(0.05) and z(0.05).

   b) Discuss the stability of Runge-Kutta method of order two and four. **(7+7)**

3. a) Derive the three-step Adams-Bashforth method for $\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$.

   b) Solve $\frac{d^2y}{dx^2} = x + y$ with y(0) = 0, y(1) = 1 by finite difference method taking $\Delta x = 0.25$ one iteration by Gauss Seidel method. **(7+7)**

4. Describe the shooting method for the solution of higher order differential equations. Hence, solve $\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y$ subject to y(0) = 0 ; y(0.2) = 1. **14**

5. a) Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$; $0 \le x \le 1, 0 \le y \le 1$ with u = 0 on the boundary of unit square, choosing $\Delta x = \Delta y = \frac{1}{3}$.

   b) Discuss the stability of the Schmidt method applied to one dimensional heat equation. **(8+6)**

6. Solve the IBVP $\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}$; $0 \le x \le 1, t \ge 1$ subject to $u(x, 0) = x(1 - x)$; $0 \le x \le 1$

$\left.\begin{matrix} u(0,1)=0 \\ u(1,t)=0 \end{matrix}\right\} t \ge 0$ by Crank-Nicolson method. Choose $\Delta x = \frac{1}{4}$ and $\Delta t = \frac{1}{64}$ and obtain a solution at the first time level and discuss the stability of Crank Nicolson Implicit method. **14**

7. a) Solve one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ explicity with conditions $u(x, 0) = \sin\pi x$, $\frac{\partial u}{\partial t}(x, 0) = 0$; $0 \le x \le 1$, $u(0, t) = 0$, $u(1, t) = 0$, $t \ge 0$ with $\Delta x = \frac{1}{4}$, $\Delta t = \frac{1}{64}$. Obtain the solution at second time level.

   b) Derive the first Lees alternating direction implicit method applied to two dimensional wave equation. **(7+7)**

8. Find a solution for $u_t = u_{xx} + u_{yy}$, $0 \le x, y \le 1, t \ge 0$ with $u(x, y, 0) = \sin\pi x \sin\pi y$ and $u = 0$ on the boundary points using alternating direction implicit method choosing $\Delta x = \Delta y = \frac{1}{3}, \Delta t = \frac{1}{72}$. **14**