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GS-321

II Semester B.A./B.Sc. Examination, May/June - 2019
CBCS (F+R) (2014-15 & onwards)

MATHEMATICS
Mathematics (Paper - 2)

Time : 3 Hours | Max. Marks : 70

Instructions : *Answer all Parts.*

PART- A

Answer **any five** sub-questions : **5x2=10**

1. (a) The binary operation * is defined on the set Z of integers by $a*b = a+b-2$ $\forall$ $a,b \in Z$. Find the identity element.

(b) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ Find fog.

(c) Find the angle between the radius vector and the tangent to the curve $r = a\theta$.

(d) Find the length of the polar subnormal to the curve $r = a\cos 2\theta$ at $\theta = \frac{\pi}{3}$.

(e) Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$

(f) Write the formula to find the length of an arc of the curve $y = f(x)$ from $x = a$ to $x = b$.

(g) Find the integrating factor of :

$$\frac{dy}{dx} + \frac{2}{x}y = x\log x$$

(h) Find the general solution of $y = px + \frac{p}{p-1}$

P.T.O.

PART - B

Answer **one** full question : **1x15=15**

2. (a) Show that the set of all fourth roots of unity forms an abelian group under multiplication.

(b) Prove that the inverse of an element in a group is unique.

(c) Show that the set {0, 3, 6, 9} is a subgroup of $G=\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ under addition modulo 12.

OR

3. (a) Let G be the set of all non-zero rational numbers and $*$ be the binary operation on G defined by $a * b = \frac{a.b}{7}$ $\forall$ a, b $\in$ G then prove that (G, $*$) is a group.

(b) If 'a' and 'b' are any two elements of a group (G, *) then prove that $(a*b)^{-1}=b^{-1}* a^{-1}$ $\forall$ a, b $\in$ G.

(c) Prove that $G=\{2, 4, 6, 8\}$ forms an abelian group under multiplication modulo 10.

PART - C

Answer **two** full questions : **2x15=30**

4. (a) Show that the curves $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$ cut orthogonally.

(b) With usual notations prove that

(i) $p=r\sin\phi$

(ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

(c) Find the radius of curvature of the curve $x^4+y^4=2$ at the point (1, 1).

OR

5. (a) Find the angle between the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$

(b) Find the pedal equation of the curve $r^n=a^n \sin n\theta$

(c) Find the envelope of the family of lines $y = mx + \frac{a}{m}$ where 'm' is a parameter.

6. (a) Find all the asymptotes of the curve

$y^3 - x^2y + 2y^2 + 4y + 1 = 0$

(b) Find the area bounded by the cardiod $r = a(1 + \cos\theta)$

(c) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) about x-axis.

OR

7. (a) Find the centre of curvature of the curve $xy = a^2$ at (a, a)

(b) Find the position and nature of the double points of the curve.

$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$

(c) Find the surface area of the solid generated by the revolution of the curve $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis.

PART - D

Answer **one** full question : **1x15=15**

8. (a) Solve :

$$\frac{dy}{dx} + \frac{1}{1+x^2} \cdot y = \frac{\tan^{-1}x}{1+x^2}$$

(b) Verify for exactness and solve

$(x^2 - ay)\,dx + (y^2 - ax)\,dy = 0$

(c) Solve $p^2 + 2py\cot x - y^2 = 0$

OR

9. (a) Solve : $\frac{dy}{dx} - \frac{2}{x} y = \frac{y^2}{x^3}$

(b) Find the general and singular solution of $\sin px.\cos y = \cos px \sin y + p$

(c) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$ where 'a' is a parameter.

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