## I Semester B.C.A. Degree Examination, March/April 2022 (Y2K14) (CBCS) (Repeaters) COMPUTER SCIENCE BCA 105 T : Discrete Mathematics

Time : 3 Hours
Max. Marks : 100
Instruction : Answer all Sections.

## SECTION - A

I. Answer any ten of the following. Each question carries 2 marks.

1) If $A=\{1,2,3,4\}$ and $B=\{3,4,7,8\}$ find $B-A$ and $A-B$.
2) If $A=\{2,3,4,5\}$ and $B=\{0,1,2,3\}$ find $A \cap B$.
3) Define Tautology.
4) Find $x, y, z$ if $\left[\begin{array}{cc}4-y & 3 \\ x & 5\end{array}\right]=\left[\begin{array}{cc}-6 & z+2 \\ 8 & 5\end{array}\right]$.
5) Construct truth table for proposition $p \vee \sim q$.
6) Find the characteristics equation of the matrix $\left[\begin{array}{ll}1 & 2 \\ 3 & 2\end{array}\right]$.
7) Find the value of ${ }^{5} P_{2}$.
8) If $\log x_{7}+\log x_{7}^{2}+\log x_{7}^{3}=6$ find $x$.
9) Define abelian group.
10) If $\vec{a}=2 \hat{i}+3 \hat{j}-4 k, \vec{b}=3 \hat{i}-4 \hat{j}-5 \hat{k}$ find $|\vec{a}+\vec{b}|$.
11) Find the distance between the point $\mathrm{A}=(-9,6)$ and $\mathrm{B}=(-7,-3)$.
12) Find the equation of the line with slope 2 and cutting off an intercept 3 on $y$-axis.

## SECTION - B

## II. Answer any six of the following. Each question carries 5 marks.

13) Prove that $(p \wedge q) \wedge \sim(p \vee q)$ is contradiction.
14) Prove that $\sim(p \leftrightarrow) \equiv \sim[(p \rightarrow q) \wedge(q-p)]$.
15) If $\mathrm{A}=\{1,4\} \mathrm{B}=\{2,3,6\} \mathrm{C}=\{2,3,7\}$ then verify that $\mathrm{A} \times(\mathrm{B}-\mathrm{C})=(\mathrm{A} \times \mathrm{B})-(\mathrm{A} \times \mathrm{C})$.
16) Write the inverse, converse and contra-positive of the given conditional "if two angles are right angles, then they are congruent".
17) If $R \rightarrow R$ is defined by $f(x)=2 x+5$ prove that ' $f$ ' is one to one and onto.
18) Verify Cayley-Hamilton theorem for the matrix $A=\left[\begin{array}{cc}3 & 1 \\ -1 & 2\end{array}\right]$ and also find inverse.
19) If $A=\left[\begin{array}{ll}2 & 3 \\ 4 & 5\end{array}\right]$ find $A^{2}-7 A-21$.
20) Solve the equation $5 x+2 y=4,7 x+3 y=5$ using matrix method.

## SECTION - C

III. Answer any six of the following. Each question carries 5 marks.
21) If $\log \left(\frac{a-b}{5}\right)=\frac{1}{2}(\log a+\log b)$ show that $a^{2}+b^{2}=27 a b$.
22) Find ' r ' if ${ }^{15} \mathrm{P}_{\mathrm{n}-1}:{ }^{16} \mathrm{P}_{\mathrm{r}-2}=3: 4$.
23) Find the number of ways in which 8 boys and 5 girls can be arranged in a row so that no two girls are together.
24) Prove that the set $G=\{1,-1, i,-i\}$ from an abelian group under multiplication.
25) Show that the set of all cubeth roots of unity from a group under multiplication.
26) If $\vec{a}=\hat{i}-2 \hat{j}+3 \hat{k}$ and $\vec{b}=2 \hat{i}+3 \hat{j}-5 \hat{k}$ find $\vec{a} \times \vec{b}$ verify that $\vec{a}$ and $(\vec{a} \times \vec{b})$ are perpendicular to each other.
27) Prove that $\vec{a} \times(\vec{b} \times \vec{c})+\vec{b} \times(\vec{c} \times \vec{a})+\vec{c} \times(\vec{a} \times \vec{b})=0$.
28) If vector $2 \hat{i}-3 \hat{j}+m \hat{k}, 2 \hat{i}+\hat{j}-\hat{k}$ and $6 \hat{i}-\hat{j}+2 \hat{k}$ are coplanar. Find $m$.
SECTION - D
IV. Answer any four of the following. Each question carries 5 marks.
29) Show that the points $(3,2)(0,5)(-3,2)$ and $(0,-1)$ are vertices of a square.
30) Find the ratio in which the $x$-axis divides the line segment joining the points $(7,-3)$ and $(5,2)$.
31) Find the equation of the straight line which passes through the point of intersection of the lines $3 x+y-10=0$ and $x+7 y-10=0$ and parallel to the line $4 x-3 y+1=0$.
32) Find the equation of the locus of the point which moves such that it is equidistant from the points $(1,2)$ and $(-2,3)$.
33) Find the value of $K$ if the lines
i) $3 x+2 y+1=0$ and $K x+2 y-1=0$ are parallel.
ii) $5 x-4 y+8=0$ and $4 x+K y+3=0$ are perpendicular.
34) Prove that the points $(2,2)$ and $(-3,3)$ are equidistant from the line $x+3 y-7=0$ and are on either side of the line.

