MSc-Mathematics - 6

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I Semester M.Sc. Examination, February - 2020
(CBCS-Y2K17/Y2K14 Scheme)

MATHEMATICS

M101T : Algebra - I

Time : 3 Hours | Max. Marks : 70

Instructions : *(i) Answer **any 5** questions.*
*(ii) **All** questions carry **equal** marks.*

1. (a) If N and M are normal subgroups of G, then prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$. **6+4+4**

(b) Show that $T : G \rightarrow G$ defined by $T(x) = x^{-1}$ is an automorphism if and only if G is abelian.

(c) For the symmetric group S_3, prove that Aut $S_3 \approx$ Inn S_3, where Aut S_3 is group of automorphisms of S_3 and Inn S_3 is group of inner automorphisms of S_3.

2. (a) Let G be a finite group and S is a finite G-set. If $x \in S$, then show that $o(G_x) = o(G)/o(\text{stab}(x))$. **5+4+5**

(b) Derive the class equation for finite groups.

(c) Define a p-group. If G is a finite group of prime power order. Prove that G has a non-trivial center.

3. (a) Show that the number of p-sylow subgroups of G, for a given prime, is congruent to 1 modulo p. **6+4+4**

(b) Let $O(G) = pq$, where p and q are distinct primes with $p < q$ and $q \not\equiv 1 \pmod p$. Then prove that G is abelian and cyclic.

(c) Show that every group of order $11^2.13^2$ is abelian.

4. (a) Define a solvable group. Prove that every subgroup of a solvable group is solvable. Further, show that symmetric group S_4 is solvable, but not simple. **7+7**

(b) State and prove the Jordan-Holder Theorem.

P.T.O.

5. (a) If R is a ring with unity in which $\{0\}$ and R are the only two left ideals, then prove that R is a division ring. **4+10**

(b) Let R and R' be two rings and ϕ be a homomorphism of R onto R' with Kernel $\mho$. Then show that :

(i) $R' \approx R/\mho$

(ii) There is one to one correspondence between the set of ideals W' of R' and the set of ideals W of R containing $\mho$.

(iii) $R/W \approx R'/W'$

6. (a) Define **2+6+6**

(i) Principal ideal of a ring

(ii) Prime ideal of a ring

(b) Define a maximal ideal of a ring R. If R is a commutative ring with unity and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field.

(c) Show that the quotient field is the smallest field containing D, where D is an integral domain.

7. (a) Show that the ring Z[i] of Gaussian integer is an Euclidean Ring. **4+4+6**

(b) Show that every Euclidean ring is a principle ideal ring.

(c) If p is a prime number of the form $4n+1$, prove that $p=a^2+b^2$ for some integers a, b.

8. (a) State and prove Gauss Lemma. **6+8**

(b) State and prove the Eisenstein criterian for the irreducibility of a polynomial with integer coefficients over the rationals.

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