No. of Printed Pages : **2**

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I Semester M.Sc. Examination, February - 2020
(CBCS-Y2K17/Y2K14 Scheme)

MATHEMATICS

M103T : Topology - I

Time : 3 Hours

Max. Marks : 70

Instructions : *(i) Answer **any five** questions.*
*(ii) **All** questions carry **equal** marks.*

1. (a) Show that if X is an infinite set and $x_0 \in X$ then $X-\{x_0\}$ is also an infinite set.

4+4+6

(b) Prove that a set X is finite if and only if X=Q or X is in one-one correspondence with $N_k=\{1, 2, ..., k\}$ for same $k \in \mathbf{N}$.

(c) Define a denumerable set. Prove that every subset of a denumerable set is either finite or denumerable.

2. (a) State and Prove Schroder-Bernstein theorem.

8+6

(b) With usual notations Prove $2^{\aleph_0} = c$.

3. (a) If (X, d) is a metric space then show that $P(x, y)=\dfrac{d(x, y)}{1+d(x, y)}$, $\forall\ x, y \in X$ is a metric space.

5+5+4

(b) Show that in a metric space the following are equivalent :

(i) $X-A$ is open

(ii) $d(A) \subseteq A$ for any $A \subseteq X$

(c) Show that in a metric space, every convergent sequence has a unique limit.

4. (a) State and Prove Baire Category theorem.

8+6

(b) Show that C(X, **R**), the collection of all continuous bounded mappings from a metric space X into **R** is a complete metric space.

P.T.O.

5. (a) Prove that every superset of a neighbourhood of a point is a neighbourhood and intersection of two neighbourhoods is a neighbourhood. **5+5+4**

(b) Let A be any subset of (X, Y). Then Prove that $A^0 = \left(\overline{A'}\right)'$, where $A' = X - A$.

(c) Prove for any subsets A and B of a topological space (X, Y),

$\overline{A \cup B} = \overline{A} \cup \overline{B}$

6. (a) Prove that a function $f: X \to Y$ is continuous at x if and only if for every neighbourhood V of $f(x)$, $f^{-1}(x)$ is a neighbourhood of x. Further, prove that $f: X \to Y$ is continuous if and only if inverses of open sets are open. **8+6**

(b) Show that a bijective function $f: X \to Y$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$ for any set $A \subseteq X$.

7. (a) State and Prove Pasting lemma. **5+5+4**

(b) Show that the closure of a connected set is connected.

(c) Show that continuous image of a connected set is connected.

8. (a) Define a Path connected space. Show that a path connected space is always connected. **5+4+5**

(b) Prove that a continuous image of a path connected space is path connected.

(c) Define Components. Prove that components are always closed.

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