PG – 650

I Semester M.Sc. Degree Examination, August/September 2021
(CBCS – Y2K17/Y2K14)
MATHEMATICS
M105T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five (5) full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Write a short note on methods of proof and disproof. Prove or disprove the following statement :
"The sum of two prime numbers each larger than two is not a prime number".

b) Using the concept of quantifiers and rules of inference, find whether the following is a valid statement.
If a triangle has two equal sides, then it is isosceles.
If a triangle is isosceles, then it has two equal sides.
The triangle ABC does not have two equal angles.
∴ The triangle ABC does not have two equal sides.

c) Show that $r \wedge (p \vee q)$ is a conclusion from the premises $p \vee q$, $q \to r$, $r \to s$ and $\sim s$.

(5+5+4)

2. a) Show that if 11 numbers are chosen from the set {1, 2, 3, ..., 20}, one of them is a multiple of another.

b) A company appoints 16 software engineers, each of whom to be assigned to one of four offices of the company. Each office should get one of these engineers. In how many ways can these assignments be made ?

c) Find the number of ways of placing 25 people into three rooms with at least one person in each room.

(4+5+5)

3. a) Model the "Tower of Hanoi" problem as a recurrence relation and solve it explicitly.

b) Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$, with initial conditions $a_0 = 2$, $a_1 = 1$.

c) The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after two days.

(4+5+5)

P.T.O.

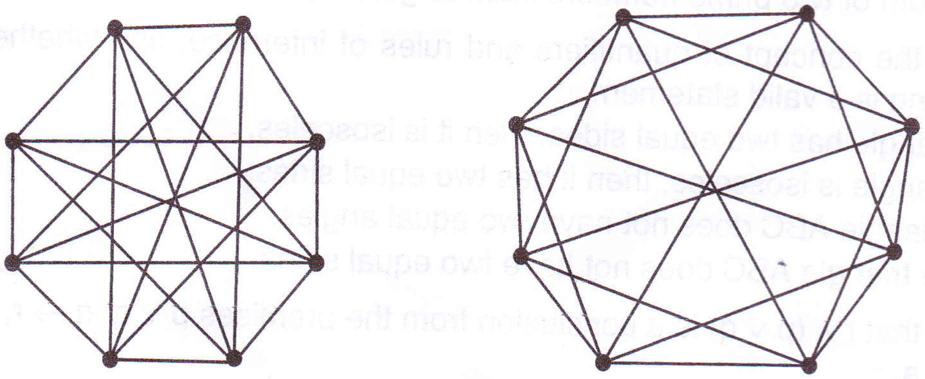
4. a) Let A = {lines in a plane}. A relation R can be defined on A such that xRy if and only if x is perpendicular to y. Determine the nature of R.

   b) Write step by step procedure of Warshall's algorithm and apply it to the following relation, R = {(1, 2), (2, 3), (3, 4), (2, 1)} on the set A = {1, 2, 3, 4}.

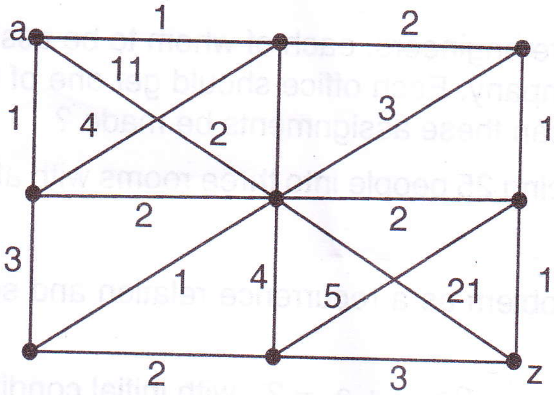
   c) Write the Hasse diagram for the set D_{20} which consists of all divisors of 20 and R = {(x, y) : x, y $\in D_{20}$ and x divides y}. **(4+5+5)**

5. a) Define a bipartite graph G. Prove that a graph G is bipartite if and only if it has no odd cycle.

   b) Define graph isomorphism. Check whether the following graphs are isomorphic or not.



   c) Find the shortest distance path between 'a' and 'z' using Dijkstra's algorithm for the following graph. **(4+5+5)**



6. a) Define an Eulerian graph. Prove under which conditions the complete graph K_p has an Eulerian cycle. Explain.

   b) What do you mean by Hamiltonicity in graphs ? Show that any k-regular simple graph with 2k – 1 vertices is Hamiltonian.

   c) Write a short note on Travelling salesman problem and illustrate. **(4+5+5)**

7. a) Define a planar graph. If G is a planar, connected (p, q) graph without triangles, then prove that $q \leq 2p - 4$, for all $p \geq 3$.

   b) With standard notations prove the following
   $k(G) \leq \lambda(G) \leq \delta(G)$.

   c) Show that for any non-trivial connected graph G, $\alpha_0(G) + \beta_0(G) = p$. **(5+5+4)**

8. a) Define a binary tree. If a binary tree has p vertices of which k are pendant, then prove that it has $p - k - 1$ vertices of degree 3.

   b) What do you mean a spanning tree ? Prove that a graph is connected if and only if it contains a spanning tree.

   c) Write and apply Prim's algorithm to find a minimum spanning tree for the weighted graph given below. **(5+4+5)**