PG – 651

I Semester M.Sc. Degree Examination, Aug./Sept. 2021
(CBCS) (Y2K17/Y2K14)
MATHEMATICS
M107SC : Mathematical Analysis

Time : 3 Hours   Max. Marks : 70

Instructions : *i) Answer **any five (5) full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Show that closed subsets of compact sets are compact.

   b) Show that every infinite bounded set has a limit point.

   c) Show that a subset E of the real line R is connected if and only if it has the following property.

   If $x \in E$, $y \in E$ and $x < z < y$ then $z \in E$. **(4+5+5)**

2. a) Let f and g be complex continuous functions on a metric space. Then prove that fg and f/g are continuous on X where $g \neq 0$.

   b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f(X) is compact. **(7+7)**

3. a) Let f be a real differentiable function on [a, b] and $f'(a) < \lambda < f'(b)$. Then show that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

   b) State and prove the Taylor's theorem. **(7+7)**

4. a) Show that $f(x) = x \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$ is continuous but not derivable at $x = 0$.

   b) State and prove first mean value theorem. **(7+7)**

5. a) If p > 0, then show that $\lim_{n\to\infty} \sqrt[n]{p} = 1$.

   b) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

   c) S.T e is irrotational. **(4+6+4)**

P.T.O.

6. a) Show that a positive term series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$.

   b) Let Σu_n and Σv_n be two positive term series and there exists a positive integer m such that $\frac{u_n}{u_{n+1}} \geq \frac{v_n}{1+v_n} \forall n \geq m$.

   Then prove the following :

   i) Σu_n is convergent, if Σv_n is convergent.

   ii) Σv_n is divergent, if Σu_n is divergent. **(7+7)**

7. a) Show that every absolutely convergent series is convergent.

   b) State and prove Logarithmic test.

   c) State and prove Merten's Theorem. **(4+4+6)**

8. a) State and prove Abel's test.

   b) Show that the Cauchy product of the series $\sum_{1}^{\infty} \frac{1}{n^2}$ with itself converges to $\frac{\pi^4}{36}$. **(7+7)**

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