PG – 649

I Semester M.Sc. Degree Examination, August/September 2021
(CBCS – Y2K17/Y2K14)
MATHEMATICS
M – 104T : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five full** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Show that $\{\psi_j(x) ; j = 1 \text{ to } n\}$ forms a fundamental set on $L_n y = 0$ on I if and only if $W\{\psi_j(x) ; j = 1 \text{ to } n\} \neq 0, \forall x \in I$.

   b) If the Wronskian of $\varphi_1(x)$ and $\varphi_2(x)$ is $3e^{4x}$ and if $\varphi_1(x) = e^{2x}$ then find $\varphi_2(x)$. **(9+5)**

2. a) Verify Lagrange's identity for $x^2y'' + 9xy' + 12y = 0$.

   b) Using the method of variation of parameters solve $x^2y'' - xy' - 3y = x^3$. **(7+7)**

3. a) State and prove Sturm's comparison theorem.

   b) Find the eigenvalues and eigenfunctions of $y'' + \lambda y = 0; y(0) = 0 = y(\pi)$. **(8+6)**

4. a) Show that the eigenvalues of a Sturm-Liouville problem are simple.

   b) Construct the Green's function for $y'' + \lambda y = x ; y(0) = 0 = y(1)$. **(7+7)**

5. a) Find the Frobenius series solution of the differential equation $2x^2y'' + xy' + (x^2 - 3)y = 0$ $(0 < x < R)$ about a regular singular point.

   b) Obtain general solution of Hermite differential equation. **(8+6)**

6. a) Prove that the Chebyshev polynomials are orthogonal over [–1, 1].

$$\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}}\,dx = \begin{cases} 0 & \text{if} \quad m \neq n \\ \pi/2 & \text{if} \quad m = n \neq 0 \\ \pi & \text{if} \quad m = n = 0 \end{cases}$$

   b) Obtain the solution of the Gauss-Hypergeometric differential equation.
   $(x - x^2)y'' + (\gamma - (\alpha + \beta + 1)x)y' - \alpha\beta\gamma\, y = 0$ at $x_0 = 1$. **(6+8)**

P.T.O.

7. a) Find the fundamental matrix and general solution for the system $\underset{\sim}{x}'(t) = A\,\underset{\sim}{x}(t)$,

$$\text{where } A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}.$$

b) Write a short note on critical points. **(10+4)**

8. a) Locate the critical point and find the nature and stability of the system

i) $\frac{dx}{dt} = x + y, \frac{dy}{dt} = 3x - y$.

ii) $\frac{dx}{dt} = 2x - 2y + 11, \frac{dy}{dt} = 11x - 8y + 49$.

b) Determine the stability of the critical point (0, 0) of the system by constructing the Liapunov function $\frac{dx}{dt} = -x + y^2, \frac{dy}{dt} = -y + x^2$. **(9+5)**
