I Semester M.Sc. Examination, August/September 2021
(CBCS – Y2K17/Y2K14)
MATHEMATICS
M – 103T : Topology – I

Time : 3 Hours

Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Show that every subset of a denumerable set is either finite or denumerable.
   b) Show that the closed interval [0, 1] is uncountable.
   c) With usual notions prove that
      i) $N_0 + N_0 = N_0$
      ii) $N_0 + C = C$. **(5+5+4)**

2. a) State and prove Schroder-Bernstein theorem.
   b) Let P(A) be the power set of a set A. Then prove that $|P(A)| = 2^{|A|}$.
   c) Define :
      i) Continuum hypothesis and
      ii) Zorn's lemma. **(7+5+2)**

3. a) State and prove Baire category theorem.
   b) State and prove contraction mapping theorem. **(8+6)**

4. a) Let X be a non-empty set and define a mapping $d : X \times X \to \mathbb{R}$ as follows

$$d(x, y) = \begin{cases} 0, & \text{when } x = y \\ 1, & \text{when } x \neq y \end{cases} \quad \forall x, y \in X.$$ Then show that d is a metric on X.

   b) Show that every bounded infinite set has a limit point.
   c) Prove that every convergent sequence is a Cauchy sequence. **(4+6+4)**

5. a) Prove that an isometry is a homeomorphism but not conversely.
   b) Prove that every metric space has a completion. **(7+7)**

6. a) Let (X, τ) be a topological space and let A and B be two subsets of X. Then prove that

   i) $d(\phi) = 0$

   ii) $A \subset B \Rightarrow d(A) \subset d(B)$

   iii) $d(A \cup B) = d(A) \cup d(B)$.

   b) Prove that a function $f : X \to Y$ is continuous at x if and only if for every neighbourhood V of f(x), $f^{-1}(x)$ is a neighbourhood of x. Further, prove that $f : X \to Y$ is continuous if and only if inverses of open sets are open. **(7+7)**

7. a) Show that a set $A \subseteq R$ is connected if and only if it is an interval in R.

   b) Prove that the union of any family $\{C_\lambda\}$ of connected sets having a non-empty intersection is connected.

   c) Show that continuous image of a connected set is connected. **(6+5+3)**

8. a) Define locally connected space. Give an example to show that a connected space is not locally connected.

   b) Show that a continuous image of a path connected space is path connected.

   c) Show that closure of a connected set is connected. **(5+5+4)**
